Exercise 1. Let F be a field and $f(x), g(x) \in F[x]$ be nonzero polynomials. Suppose that $f(x)$ and $g(x)$ are *relatively prime*, i.e. that if $h(x) \in F[x]$ divides both $f(x)$ and $g(x)$ then $h(x)$ is a nonzero constant. Prove that there exist $r(x), s(x) \in F[x]$ so that $r(x)f(x) + s(x)g(x) = 1$.

Exercise 2.(Partial Fractions I) Let F be a field and $c(x), f(x), g(x) \in F[x]$ be nonzero polynomials. Prove that if $f(x)$ and $g(x)$ are relatively prime and $\deg c(x) < \deg f(x)g(x)$ then there exist $r(x), s(x) \in F[x]$ with $\deg r(x) < \deg f(x)$ and $\deg s(x) < \deg g(x)$ so that

$$\frac{c(x)}{f(x)g(x)} = \frac{r(x)}{f(x)} + \frac{s(x)}{g(x)}$$

in $F(x)$.

Exercise 3.(Partial Fractions II) Let F be a field and $c(x), f(x) \in F[x]$ be nonzero polynomials. Prove that if $\deg c(x) < \deg f(x)^2$ then there exist $r(x), s(x) \in F[x]$ with $\deg r(x), \deg s(x) < \deg f(x)$ so that

$$\frac{c(x)}{f(x)^2} = \frac{r(x)}{f(x)^2} + \frac{s(x)}{f(x)}$$

in $F(x)$.