Exercise 1. Let $D$ be an integral domain. Prove that if $p_1, p_2, \ldots, p_m, q_1, q_2, \ldots, q_n \in D$ ($m, n \in \mathbb{Z}^+$) are primes and $p_1p_2\cdots p_m = q_1q_2\cdots q_n$ then $m = n$ and, after possibly reordering, $p_i$ and $q_i$ are associates for $i = 1, 2, \ldots, m$. [Suggestion: Induct on $m$.]

Exercise 2. Let $R$ be a ring and let $I_1 \subset I_2 \subset I_3 \subset \cdots$ be an ascending chain of ideals in $R$ and let

$$I = \bigcup_{j=1}^{\infty} I_j.$$

a. Prove that $I$ is an ideal in $R$.

b. If $R$ has an identity and each $I_j$ is proper, prove that $I$ is also proper.

Exercise 3. Page 335, # 38

Exercise 4. Page 340, # 24