Modern Algebra II
Spring 2007

Roots of Polynomials

Exercise 1. Let F be a field of characteristic $p \neq 0$. Prove that $x^{p^n} - x \in F[x]$ does not have multiple zeros in any extension of F.

Exercise 2. Prove that $f(x)^p = f(x^p)$ for any $f(x) \in \mathbb{Z}_p[x]$ (p a prime).

Exercise 3. Prove that for any prime p the polynomial $g(x) = x^p - x + 1$ is irreducible over $\mathbb{Z}_p$. [*Hint:* Start by showing that if α is a root of $g(x)$ in some extension of $\mathbb{Z}_p$ then so is $\alpha + 1$.]