Exercise 1. Given a nonnegative $\theta_0 \in \mathbb{R}$ let W_{θ_0} denote the wedge-shaped region defined by

$$W_{\theta_0} = \{re^{i\theta} \in \mathbb{C} \,|\, r \geq 0, \theta \in [0, \theta_0]\}.$$

a. Sketch the region W_{θ_0} for a few values of $\theta_0 \in [0, 2\pi]$. Which value of θ yields the first quadrant? The upper half-plane? The entire complex plane?

b. Let $n \in \mathbb{Z}$, $n \geq 2$ and $\theta_0 \in [0, 2\pi/n)$. Prove carefully that the function $f(z) = z^n$ maps the region W_{θ_0} one-to-one and onto the region $W_{n\theta_0}$.

c. In part (b), if we allow $\theta_0 = 2\pi/n$, then $f(z)$ is no longer both one-to-one and onto. Is it either one of these? Why?