

In the following exercises, $f(x)$ denotes a real-valued function defined on $[a, \infty)$ for some $a \in \mathbb{R}$.

Exercise 1. Prove that $\limsup_{x \rightarrow \infty} |f(x)| = 0$ implies that $\lim_{x \rightarrow \infty} f(x) = 0$.

Exercise 2. Prove that if $\alpha = \limsup_{x \rightarrow \infty} f(x)$ then for any $\epsilon > 0$ there is an $x_0 \geq a$ so that $f(x) < \alpha + \epsilon$ for all $x \geq x_0$. Prove that this converse of this statement is false.