In the following exercises, \( f(x) \) denotes a real-valued function defined on \([a, \infty)\) for some \( a \in \mathbb{R} \).

**Exercise 1.** Prove that \( \limsup_{x \to \infty} |f(x)| = 0 \) implies that \( \lim_{x \to \infty} f(x) = 0 \).

**Exercise 2.** Prove that if \( \alpha = \limsup_{x \to \infty} f(x) \) then for any \( \epsilon > 0 \) there is an \( x_0 \geq a \) so that \( f(x) < \alpha + \epsilon \) for all \( x \geq x_0 \). Prove that this converse of this statement is false.