Exercise 1. Let $n \in \mathbb{N}$, $n \geq 2$. Recall that if $f \in \widehat{\mathbb{Z}/n\mathbb{Z}}$ then there is a unique $k \in \{0, 1, \ldots, n-1\}$ so that $f(\overline{1}) = e^{2k\pi i/n}$ and that we used this fact to define $\phi : \widehat{\mathbb{Z}/n\mathbb{Z}} \to \mathbb{Z}/n\mathbb{Z}$ by $\phi(f) = \overline{k}$. Prove that ϕ is an isomorphism.

Exercise 2. Let G and H be groups.

- **a.** Let $f \in \widehat{G \times H}$. For $a \in G$, $b \in H$ define $f_G(a) = f(a, e)$ and $f_H(b) = f(e, b)$. Prove that $f_G \in \widehat{G}$ and $f_H \in \widehat{H}$.
- **b.** Let $f_1 \in \widehat{G}$ and $f_2 \in \widehat{H}$ and define $f(a, b) = f_1(a)f_2(b)$ for $(a, b) \in G \times H$. Prove that $f \in \widehat{G \times H}$ and that $f_G = f_1$ and $f_H = f_2$.

These two facts complete the proof that $\widehat{G \times H} \cong \widehat{G} \times \widehat{H}$.