Let G be a finite group. Recall we defined

$$V_G = \{f : G \to \mathbb{C}\},$$

which is a $|G|$-dimensional vector space over $\mathbb{C}$ under the usual pointwise addition and scalar multiplication of functions. For $f, g \in V_G$ we defined their inner product to be

$$\langle f, g \rangle = \sum_{a \in G} f(a)\overline{g(a)}.$$

We further define

$$W_G = \{f : G \to \mathbb{C} \mid f(aba^{-1}) = f(b) \text{ for all } a, b \in G\},$$

the subspace of *class functions*.

Exercise 1. Prove that $W_G = V_G$ if and only if G is abelian.

Exercise 2. Prove that $\widehat{G} \subset W_G$. Conclude that if $|G| = |\widehat{G}|$ then G is abelian.

Exercise 3. Apostol, p 144, #13.