

Exercise 1. Let $k \in \mathbb{N}$ and $c : \mathbb{N} \rightarrow \mathbb{C}$ be an arithmetic function that has period k , is completely multiplicative, and vanishes on the set $\{n \in \mathbb{N} \mid (n, k) > 1\}$. Prove that c is a Dirichlet character.

Exercise 2. Let $d, k \in \mathbb{N}$ with $d \mid k$. Prove that if χ and ψ are Dirichlet characters mod k and d , respectively, then $\psi\chi$ is a Dirichlet character mod k .¹

Exercise 3. Apostol, p 175, #5.

¹The decomposition $\chi = \chi_1\chi_0$ arising from an induced modulus is a special case of this phenomenon.