

Exercise 1. If χ_0 is the principal character mod k , prove that

$$G(n, \chi_0) = \sum_{d|(n,k)} \mu\left(\frac{k}{d}\right) d.$$

Exercise 2. Show that

$$\sum_{n=0}^{\infty} \frac{1}{(4n+1)(4n+3)} = \frac{\pi}{8}.$$

[*Suggestion:* Express the sum in question in terms of $L(1, \chi)$ for some Dirichlet character χ .]

Exercise 3. Let χ be the nonprincipal Dirichlet character mod 12 that satisfies $\chi(5) = 1$. Evaluate $L(1, \chi)$.

Exercise 4. If χ is a Dirichlet character mod k that satisfies $\chi(-1) = -1$ prove that

$$\sum_{m=1}^{k-1} \chi(m) \log \sin\left(\frac{\pi m}{k}\right) = 0.$$