Number Theory II Fall 2008

Assignment 5

**Exercise 1.** Prove that

$$\sum_{n \le x} \frac{\log n}{n} = \frac{1}{2} \log^2 x + O(1).$$

**Exercise 2.** p 70, # 2

**Exercise 3.** Prove that for any  $k \in \mathbb{N}$  and any  $s > 0, s \neq 1$ 

$$\sum_{\substack{n \le x \\ (n,k)=1}} \frac{1}{n^s} = \frac{\varphi(k)}{k} \frac{x^{1-s}}{1-s} + \zeta(s) \prod_{p|k} \left(1-p^{-s}\right) + O\left(x^{-s}\right).$$

On what does the implied constant in the error term depend?