Number Theory II
Fall 2008

Assignment 7

Exercise 1. Let a be a nonnegative real number and f a function that is Riemann integrable on every finite subinterval of $[a, \infty)$. Let

$$\overline{f}(x) = \frac{1}{x} \int_a^x f(t)\, dt$$

for $x \in [a, \infty)$, $x > 0$. Prove that $\lim_{x \to \infty} f(x) = 0$ implies that $\lim_{x \to \infty} \overline{f}(x) = 0$.

Exercise 2.

a. Prove the analogue of the statement of Exercise 1 with the limit 0 replaced by an arbitrary L. [*Hint:* Consider the function $f(x) - L$.]

b. Show that the converse of the statement in Exercise 1 is false.