Number Theory II Fall 2008

Assignment 9

Exercise 1. Let $\Lambda_2 = \Lambda \cdot \log + \Lambda * \Lambda$. Prove that Selberg's asymptotic formula (Theorem 4.18) is equivalent to the statement

$$\sum_{n \le x} \Lambda_2(n) = 2x \log x + O(x).$$

Exercise 2. Let g(x) and h(x) be continuous at x = 0 with g(0) = h(0) = L. Suppose f(x) is a function with the following property: for any $\epsilon > 0$ there exists an x_0 so that $g(\epsilon) \leq f(x) \leq h(\epsilon)$ for all $x \geq x_0$. Prove that $\lim_{x\to\infty} f(x) = L$. What choices of g and h are used in Levinson to prove the prime number theorem (see page 244)?