Number Theory I — Assignment 1

Spring 2012 — Due January 19

Exercise 1. Show that the square of any integer must have the form $3k$ or $3k+1$. [*Hint:* Given $n \in \mathbb{Z}$, according to the Division Algorithm $n = 3q + r$ where $r = 0, 1$ or 2. Consider what happens when you square n in each case.]

Exercise 2. Use the preceding exercise to prove that there are no integer solutions to the equation $3a^2 - b^2 = 1$.

Exercise 3. Show that the cube of any integer has the form $7k$ or $7k \pm 1$.

Exercise 4. Argue as in Exercise 1 to show that for any integer a:

a. $2|a(a+1)$.

b. $3|a(a+1)(a+2)$.

Exercise 5. Show that if a and b are odd, then $8|(a^2 - b^2)$.

Exercise 6. Show that $d = \gcd(6a + 5b, 10a + 8b)$ is a common divisor of $2a$ and b. Is it true that $d|a$?