Number Theory I
Spring 2012

Assignment 2.1
Due January 24

Exercise 1. Show that the product of any three consecutive integers is divisible by 6. [*Suggestion:* Use Exercise 4 from the previous assignment.]

Exercise 2. Prove that if $n \geq 0$, then $(3n)!/(3!)^n$ is an integer.

Exercise 3. Prove the following properties of the greatest common divisor.

a. If $\gcd(a, b) = 1$ and $\gcd(a, c) = 1$, then $\gcd(a, bc) = 1$. [*Suggestion:* Express the first two gcds as linear combinations, then multiply these expressions together.]

b. If $\gcd(a, b) = 1$ and $c|a$, then $\gcd(c, b) = 1$.

c. If $\gcd(a, b) = 1$, $d|ac$ and $d|bc$, then $d|c$.

d. If $a|bc$ then $a| \gcd(a, b) \gcd(a, c)$. [See the suggestion for part **a**.]

Exercise 4. Let a, b be nonzero and let k be positive. Prove that $\operatorname{lcm}(ka, kb) = k \operatorname{lcm}(a, b)$. [*Suggestion:* Use the relationship between the gcd and the lcm.]

Exercise 5. Prove that if a and b are positive, then the following are equivalent.

a. $a|b$

b. $\gcd(a, b) = a$

c. $\operatorname{lcm}(a, b) = b$

Exercise 6. Let a and b be nonzero. Prove that $\operatorname{lcm}(a, b)$ divides every common multiple of a and b. [*Suggestion:* Show that when a common multiple n is divided by $\operatorname{lcm}(a, b)$, the remainder in the Division Algorithm is also a common multiple.]