Number Theory I — Spring 2012

Assignment 2.2 — Due January 24

Exercise 1. Show that if a and b are nonzero and $ax + by = \gcd(a, b)$, then x and y are relatively prime.

Exercise 2. Recall we proved in class that if $q_1, q_2, \ldots, q_n$ are the quotients occurring in the computation of $\gcd(a, b)$ via the Euclidean algorithm, and x and y are defined by

$$\begin{pmatrix} x & y \\ * & * \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & -q_n \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & -q_{n-1} \end{pmatrix} \cdots \begin{pmatrix} 0 & 1 \\ 1 & -q_1 \end{pmatrix}$$

then $ax + by = \gcd(a, b)$. Use the Euclidean algorithm and this supplementary result to find $\gcd(a, b)$ and express it as a linear combination of a and b for the following pairs of integers.

a. 341, 297

b. 840, 361

c. 1769, 2378

Exercise 3. Find the least common multiples of the pairs in the previous exercise.

Exercise 4. Prove that if $\gcd(a, b) = 1$, then $\gcd(a + b, ab) = 1$.

Exercise 5. Prove that $\gcd(a, b, c) = \gcd(\gcd(a, b), c)$. [*Suggestion:* Show that the set of common divisors of a, b and c is the same as the set of common divisors of $\gcd(a, b)$ and c.]

Exercise 6. Find x, y, z so that

$$\gcd(198, 288, 512) = 198x + 288y + 512z.$$

[*Suggestion:* Use the previous exercise.]