NUMBER THEORY I   ASSIGNMENT 3.1
SPRING 2012   DUE JANUARY 31

Exercise 1. Find all solutions to the following Diophantine equations.

   a. $6x + 51y = 22$

   b. $221x + 35y = 93$

   c. $158x - 57y = 7$

Exercise 2. The neighborhood theater charges \$1.80 for adult admissions and \$.75 for children. On a particular evening the total receipts were \$90. Assuming more adults that children were present, how many people attended?

Exercise 3. Prove the following statements.

   a. Any prime of the form $3n + 1$ is also of the form $6m + 1$.

   b. The only prime of the form $n^2 - 4$ is 5.

   c. The only prime of the form $n^3 - 1$ is 7.

   d. If p is prime and $3p + 1$ is a perfect square, then $p = 5$.

Exercise 4. Prove that every number of the form $n^4 + 4$ ($n \neq \pm 1$) is composite. [*Suggestion:* Factor the polynomial $x^4 + 4$.]

Exercise 5. If $n > 4$ is composite, prove that n divides $(n-1)!$. [*Suggestion:* Write $n = ab$ with $1 < a, b < n$ and think about the numbers that occur in the repeated product defining $(ab - 1)!$.]