Number Theory I
Spring 2012

Assignment 4.1
Due February 7

Exercise 1. As a generalization of the notion of twin primes, one might define a prime triplet to be a triple of integers $(p, p+2, p+4)$, all of which are prime. Prove that this definition isn't very "interesting" by showing that $(3, 5, 7)$ is the only prime triplet.

Exercise 2. A better definition of prime triplet might be a triple of the form $(p, p+2, p+6)$, for which all three entries are prime. Find five prime triplets using this definition.

Exercise 3. Find a prime divisor of $N = 4(3 \cdot 7 \cdot 11) - 1$ of the form $4n + 3$. Do the same for $N = 4(3 \cdot 7 \cdot 11 \cdot 15) - 1$.

Exercise 4. Let p_n denote the nth prime. Use induction and Bertrand's postulate to prove that for $n > 3$,

$$p_n < p_1 + p_2 + \cdots + p_{n-1}.$$

Exercise 5. Prove that there are infinitely many primes of the form $6n + 5$.

Exercise 6. Prove that for any $n \geq 2$ the arithmetic progression

$$a + b, a + 2b, a + 3b, \ldots$$

where $\gcd(a, b) = 1$, contains n consecutive composite terms. [*Suggestion:* Let $k = (a + b)(a + 2b) \cdots (a + nb)$ and consider n consecutive terms beginning with $a + (k + 1)b$.]