Partial Differential Equations — Assignment 1
Spring 2012 — Due January 17

Exercise 1. Verify that each of the following functions is a solution of the two-dimensional Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

a. $u(x,y) = x^3 - 3xy^2$ **b.** $u(x,y) = \frac{1}{2}\ln(x^2+y^2)$ **c.** $u(x,y) = e^{x^2-y^2}\cos 2xy$

Exercise 2. Let F and G be twice-differentiable functions of one variable. Show that

$$u(x,t) = F(x+ct) + G(x-ct) \tag{1}$$

is a solution to the one-dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}. \tag{2}$$

Exercise 3. Use the previous exercise to find a solution of the wave equation (2) with initial data

$$u(x,0) = \frac{1}{1+x^2}\ ,\ u_t(x,0) = 0\ ,\ -\infty < x < \infty.$$

[*Hint:* If u is given by (1), these two conditions yield a pair of equations in the unknown functions F and G. Solve these equations to determine F and G.]