PARTIAL DIFFERENTIAL EQUATIONS
SPRING 2012

ASSIGNMENT 14
DUE APRIL 26

Exercise 1. Consider the boundary value problem

$$(1-x^2)y'' - 2xy' + \lambda y = 0, \qquad -1 < x < 1,$$
$$y(-1) \text{ and } y(1) \text{ bounded.}$$

a. Show that this is a singular Sturm-Liouville problem. Give at least three reasons that it is singular (as opposed to regular).

b. Show that although this problem is singular, eigenfunctions with distinct eigenvalues are nonetheless orthogonal on $[-1, 1]$ (with respect to the appropriate weight). [*Suggestion:* Use the formula derived in class for the inner product of eigenfunctions of a S-L problem.]

Exercise 2. Repeat the preceding exercise for the boundary value problem

$$(1+x)y'' + y' + (\lambda x^3 - 3)y = 0, \qquad 0 < x < 1,$$
$$y(0) = y(1)/2, \;\; y'(0) = 4y'(1).$$