Partial Differential Equations
Spring 2012

Assignment 2.1
Due January 24

1 - 3: Solve the differential equation using the method of characteristic curves.

Exercise 1. $\cos x \dfrac{\partial u}{\partial x} + \sin x \dfrac{\partial u}{\partial y} = 0$

Exercise 2. $\dfrac{\partial u}{\partial x} - 2xy \dfrac{\partial u}{\partial y} = 0$

Exercise 3. $\dfrac{\partial u}{\partial x} + (2x + y) \dfrac{\partial u}{\partial y} = 0$

4 - 5: The next two problems concern the partial differential equation

$$\frac{\partial u}{\partial t} = -v \frac{\partial u}{\partial x} - ru, \qquad (1)$$

in which v and r are constants.

Exercise 4.

- **a.** Show that if $f(x)$ is differentiable, then $u(x,t) = e^{-rt} f(x - vt)$ is a solution to (1).
- **b.** Use a linear change of variables (as we did in class) to show that every solution to (1) has the form $u(x,t) = e^{-rt} f(x - vt)$.

Exercise 5. Let $r = 2$ and $v = 5$ in (1).

- **a.** Find the solution to (1) that satisfies the initial condition $u(x,0) = xe^{-x^2}$.
- **b.** Use Maple to plot u versus x for several different values of $t \geq 0$.[1] Explain what is happening to the graph as t increases. [*Note:* Be sure to attach a print out of your Maple code to this assignment.]

[1] This can be done in several different ways. The easiest is probably to use the `animate` command, as we did in class for the solutions to the transport equation. If you are unsure, just ask!