Partial Differential Equations	Assignment 3.2
Spring 2012	Due January 31

Exercise 1. Textbook exercises 2.1.3(b,c) and 2.1.4(b,c).

Exercise 2. Textbook exercise 2.1.15.

Exercise 3. Textbook exercises 2.1.19 and 2.1.20. If you choose to use Maple to produce the required plots, the function $[x]$ is invoked by the command `floor(x)`.

Exercise 4. Let

$$\mathbf{x}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \mathbf{x}_2 = \begin{pmatrix} -1 \\ 3 \\ 1 \\ -2 \end{pmatrix}, \mathbf{x}_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \mathbf{x}_4 = \begin{pmatrix} -2 \\ 1 \\ -3 \\ 1 \end{pmatrix}.$$

a. Check that the vectors $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$ and $\mathbf{x}_4$ are pairwise orthogonal.

b. Let $\mathcal{B} = \{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4\}$. Find the coordinates for $\mathbf{x} = (1, -2, 3, -4)$ relative to $\mathcal{B}$.

c. Repeat part **b.** for the vector $\mathbf{x} = (2, 1, 0, 3)$.

d. If $\mathbf{x} = (a, b, c, d)$, give a formula for the $\mathbf{x}_2$-coordinate of $\mathbf{x}$.

Exercise 5. Let $p_0(x) = 1$, $p_1(x) = x$, $p_2(x) = 3x^2 - 1$ and $p_3(x) = 5x^3 - 3x$.

a. Verify that p_0, p_1, p_2 and p_3 are pairwise orthogonal on the interval $[-1, 1]$.

b. Determine which of p_0, p_1, p_2 and p_3 remain orthogonal on the interval $[0, 1]$.

c. Let $p(x) = x^3 + x + 1$. Compute $a_j = \langle p, p_j \rangle / \langle p_j, p_j \rangle$ for $j = 0, 1, 2, 3$. Use the interval $[-1, 1]$ in the inner product.

d. Compute $a_0 p_0(x) + a_1 p_1(x) + a_2 p_2(x) + a_3 p_3(x)$ and compare with $p(x)$.