PARTIAL DIFFERENTIAL EQUATIONS
SPRING 2012

PROJECT 1
DUE MARCH 29

Project. Consider the problem of determining the temperature in a bar with one insulated end and one radiating end. This can be modeled using the heat equation with Robin boundary conditions as:

$$u_t = c^2 u_{xx}, \qquad 0 < x < L,\ 0 < t, \tag{1}$$

$$u_x(0,t) = 0, \qquad 0 < t, \tag{2}$$

$$u_x(L,t) = -\kappa u(L,t), \qquad 0 < t, \tag{3}$$

$$u(x,0) = f(x), \qquad 0 < x < L, \tag{4}$$

where $\kappa > 0$. In this project you will derive the solution to this problem.

1. Use separation of variables to find the normal modes satisfying (1), (2) and (3). Be sure to include a careful analysis of the possible signs of the separation constant. Also provide a graphical demonstration of the fact that there are infinitely many possible values for this constant.
2. Write down the general series solution to (1), (2) and (3) using the principle of superposition. Express the initial condition (4) as a series expansion for f.
3. Verify that the functions appearing in the series expansion for f in Part 2 are pairwise orthogonal on the interval $[0, L]$. Use this to give explicit formulas (as integrals involving f and these functions) for the coefficients appearing in this series.
4. Summarize your findings from Parts 1 – 3.
5. Create a Maple file that plots and animates the solution to (1) – (4). Your code should be carefully documented and as general as possible. I should be able to easily change the values of c, L, κ and the definition of f, without affecting the utility of the remaining code. I will grade your file by choosing my own values and then executing your code.

Additional instructions.

- You are permitted and encouraged to work in a group of up to three people.
- Written work (one copy per group) for Parts 1 – 4 must be turned in by **5 pm on March 29**. Be sure to include the names of everyone in your group, and to follow the homework guidelines given in the syllabus.
- The Maple file you create in Part 5 must be uploaded through TLEARN by **5 pm on March 29**, as well. Only one group member needs to upload the file, and the file name should include the last names of everyone in your group.