NUMBER THEORY
SPRING 2014

ASSIGNMENT 2.2
DUE JANUARY 28

Throughout the following exercises, f_n denotes the nth Fibonacci number:

$$f_0 = 0,\ f_1 = 1,\ f_{n+2} = f_{n+1} + f_n \text{ for } n \geq 0.$$

Exercise 1. Prove that for $n \geq 1$

$$\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}^n = (-1)^n \begin{pmatrix} f_{n-1} & -f_n \\ -f_n & f_{n+1} \end{pmatrix}.$$

Exercise 2. Prove that for $n \geq 1$

$$\begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}^{n-1} = (-1)^n \begin{pmatrix} f_{n-1} & -f_n \\ -f_{n+1} & f_{n+2} \end{pmatrix}.$$

Exercise 3. Prove that for $n \geq 1$

$$f_{n-1}f_{n+2} - f_n f_{n+1} = (-1)^n.$$

[*Suggestion:* Use the preceding exercises and the (matrix version of the) Euclidean Algorithm computation of (f_{n+2}, f_{n+1}).]