



NUMBER THEORY
SPRING 2014

ASSIGNMENT 3.2
DUE FEBRUARY 4

Exercise 1. Textbook exercise 1.44

Exercise 2. Textbook exercise 1.76

Exercise 3. Textbook exercise 1.83

Exercise 4. Textbook exercise 1.87

Exercise 5. The *Hilbert monoid* is the set

$$H = 1 + 4\mathbb{N}_0 = \{1 + 4k \mid k \in \mathbb{N}\}.$$

- a. Show that H is closed under multiplication, i.e. prove that if $a, b \in H$, then $ab \in H$.
- b. Define an element $a \in H$ to be *irreducible*¹ if its only divisors *in* H are 1 and itself. Prove that 9, 21, 33 and 77 are all irreducible elements of H .
- c. Show that unique factorization into irreducibles *fails* in H by finding two such factorizations of 693.

¹These are the analogues of primes in \mathbb{N} .