Math 3357 Spring 2012

Partial Differential Equations

Second Midterm Exam

Thursday, March 8

Your name (please print):

Instructions: This is a closed book, closed notes exam. You may use a calculator, **but the use of other electronic devices such as cell phones, mp3 players, etc. is not permitted.** Unless indicated otherwise, you must justify all of your answers to receive credit. Unjustified answers and/or disorganized or otherwise illegible work will receive partial credit at best. Notation is important, and points will be deducted for incorrect use. Please do all of your work on the paper provided.

The Honor Code requires that you neither give nor receive any aid on this exam.

Please indicate that you have read and understood these guidelines by signing your name in the space provided:

Pledged: ______________________________

Do not write below this line

Problem	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Points	10	10	10	14	14	12	30
Score							

Total:____________________

1. Let

$$f(x) = \begin{cases} 1 & \text{if } 0 \le x < \frac{1}{2}, \\ \frac{3}{2} - x & \text{if } \frac{1}{2} \le x \le 1. \end{cases}$$

a. Write down expressions (in terms of explicit integrals) that give the coefficients in the cosine and sine series expansions of $f(x)$ on the interval $[0, 1]$. Be sure to indicate to which expansion your coefficients belong. **Do not attempt to evaluate these expressions.**

b. Sketch the graphs of the cosine and sine series of $f(x)$ on the interval $[-3, 3]$. Be sure to carefully label your graphs.

2. Find the complex form of the Fourier series for the 2-periodic function

$$f(x) = \begin{cases} e^x & \text{if } 0 \le x < 2, \\ f(x+2) & \text{otherwise.} \end{cases}$$

3. Use separation of variables to convert the PDE

$$u_{tt} + 3u_t = u_{xx} - 2u_x + u_{yy}$$

into a system of three ODEs. **Do not attempt to solve these equations.**

4. A string is stretched along the interval $[0, 2]$ on the x-axis, giving it a c^2 value of $1/9$. The string is then deformed to an initial shape given by

$$f(x) = 2\sin\frac{\pi x}{2} - 3\sin 2\pi x$$

and is released at $t = 0$ with velocity given by

$$g(x) = 1 - x.$$

Give an expression for the shape of the string at any later time t.

5. Solve the boundary value problem

$$\begin{aligned}
&u_t = 4u_{xx}, && 0 < x < 10,\ t > 0,\\
&u(0,t) = 0, && t > 0,\\
&u(10,t) = 50, && t > 0,\\
&u(x,0) = 10x, && 0 \le x \le 10.
\end{aligned}$$

6. Circle **T** if the given statement is always true, or **F** if it is sometimes false.

a. **T / F** Every solution of the vibrating string problem is periodic in time, with period $L/2c$.

b. **T / F** The normal modes of a vibrating rectangular membrane are periodic in time, with a common period.

c. **T / F** The boundary condition $u(0,t) + u_x(0,t) = 0$ is homogeneous.

d. **T / F** The real and complex Fourier series of $f(x)$ both converge to the same function.

e. **T / F** A steady state solution to a PDE has the form $u =$ constant.

f. **T / F** The function $\sin \pi x/3$ is its own sine series expansion on the interval $[0,1]$.

7. Consider the boundary value problem

$$
\begin{aligned}
&u_t = u_{xx}, && 0 < x < 4,\ t > 0, && (1)\\
&u_x(0,t) = 0, && t > 0, && (2)\\
&u(4,t) = 0, && t > 0, && (3)\\
&u(x,0) = f(x), && 0 \le x \le 4. && (4)
\end{aligned}
$$

a. Provide a brief physical interpretation of this problem. What does the function $u(x,t)$ represent? What do the boundary and initial conditions mean?

b. Use separation of variables to find the normal modes that satisfy (1) – (3). [*Hint:* Show that the separation constant must be negative.]

c. State the orthogonality relations for the separated solutions. **Do not attempt to verify these relations.**

d. Assuming orthogonality and completeness of the separated solutions, write down the complete solution to (1) – (4). Be sure to give expressions for any coefficients that you introduce. **Do not attempt to evaluate these expressions.**