Partial Differential Equations — Assignment 1

Spring 2014 — Due January 23

Exercise 1. Show that $u(x,y) = x^3 - 3xy^2$ and $u(x,y) = e^{x^2-y^2}\cos 2xy$ are solutions of the two-dimensional Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

Exercise 2. Show that

$$u(x,t) = \frac{1}{\sqrt{4\pi c^2 t}} \exp\left(-\frac{x^2}{4c^2 t}\right)$$

is a solution of the one-dimensional heat equation $u_t = c^2 u_{xx}$.

Exercise 3. Show that $u(r,\theta) = \ln r$ and $u(r,\theta) = r\cos\theta$ are both solutions to the PDE

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0.$$

Exercise 4. Let f be a differentiable function of one variable. Show that

$$u(x,t) = f(x - vt)$$

is a solution to the one-dimensional transport equation

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial x} = 0.$$

Use this fact to find the solution that satisfies the *initial condition*

$$u(x,0) = \frac{1}{x^2+1}.$$