Partial Differential Equations
Spring 2014

Assignment 4.1
Due February 13

Exercise 1. Textbook exercise 2.1.15

Exercise 2. Textbook exercises 2.2.1–2.2.4

Exercise 3. Let

$$\begin{aligned} \mathbf{b}_1 &= (1,1,0,1), \\ \mathbf{b}_2 &= (-1,3,1,-2), \\ \mathbf{b}_3 &= (-1,0,1,1), \\ \mathbf{b}_4 &= (-2,1,-3,1). \end{aligned}$$

a. Show that the vectors $\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$ and $\mathbf{b}_4$ form an orthogonal basis for $\mathbb{R}^4$.

b. Let $\mathbf{x} = (1,-2,3,-4)$. Express $\mathbf{x}$ as a linear combination of $\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3, \mathbf{b}_4$.

c. Repeat part **b** for the vector $\mathbf{y} = (2,1,0,3)$.

d. Repeat part **b** for the vector $\mathbf{z} = (a,b,c,d)$.

Exercise 4. Let $p_0(x) = 1$, $p_1(x) = x$, $p_2(x) = 3x^2 - 1$ and $p_3(x) = 5x^3 - 3x$.

a. Show that p_0, p_1, p_2 and p_3 are pairwise orthogonal on the interval $[-1,1]$.

b. Which of p_0, p_1, p_2 and p_3 remain orthogonal on the interval $[0,1]$?

c. Because they are orthogonal on $[-1,1]$, the polynomials p_0, p_1, p_2 and p_3 form a basis for the vector space of all polynomials of degree ≤ 3. Express $p(x) = x^3 + x + 1$ as a linear combination of p_0, p_1, p_2 and p_3. [*Suggestion:* Emulate the inner product procedure used for vectors.]

d. Repeat part **c** for the polynomial $q(x) = x^3 - 2$.

Exercise 5. Show that for $m, n \in \mathbb{N}$, on the interval $[-\pi, \pi]$ one has

$$\langle \sin(mx), \sin(nx) \rangle = \begin{cases} 0 & \text{if } m \neq n, \\ \pi & \text{if } m = n. \end{cases}$$