Consider a rectangular slab of metal of width $a$, height $b$ and depth $d$ (we will specify specific units of measurement later). Suppose the slab is well-insulated along its top and bottom faces, and that we control the temperature on the remaining sides as follows. One pair of adjacent sides is kept in contact with blocks that are refrigerated to certain constant (possibly non-equal) temperatures. The other pair of sides is kept in contact with heaters that provide certain constant (but again possibly non-equal) amounts of thermal energy per unit of time (heat flux).

After a sufficient amount of time has passed, the temperature throughout the slab will stabilize. Although the entire slab is three-dimensional, the temperature $u(x, y)$ at a depth of $d/2$ can be modeled by the two-dimensional steady state heat equation, $\nabla^2 u = 0$. Our goals in this project are: 1) to find an exact analytic expression that models the temperature $u(x, y)$ at each half-depth point in the plate, and 2) to compare this theoretical model with experimental data obtained from an apparatus fitting the description above.



(a) The experimental set-up
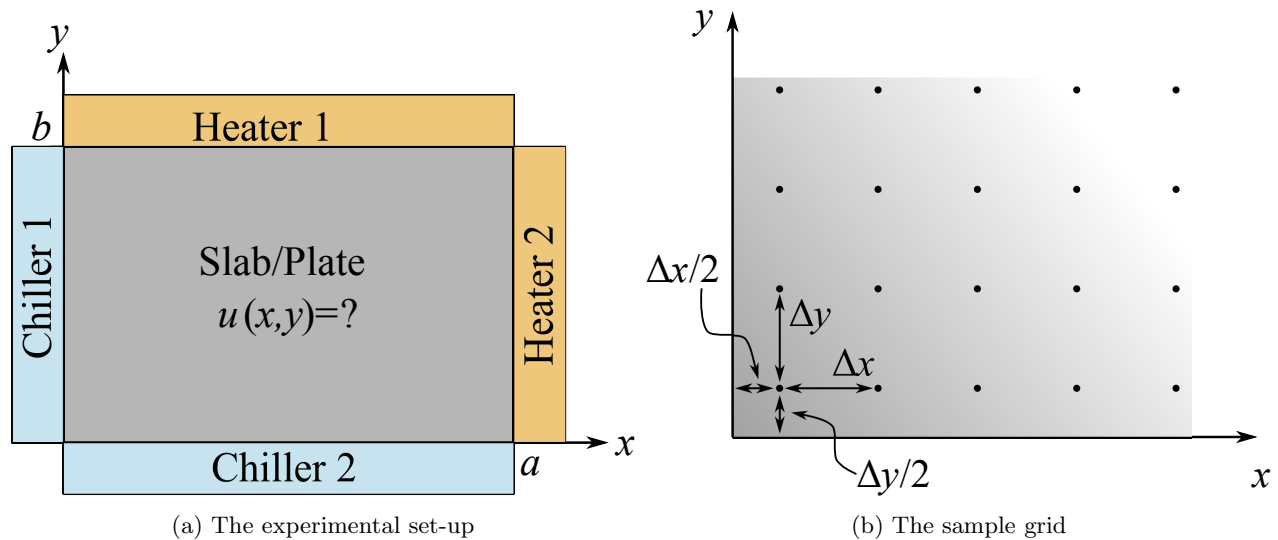
(b) The sample grid

Figure 1: Diagrams

If we impose rectangular coordinates using one of the corners of the slab as the origin, the situation we've just described (when viewed from above) is depicted in Figure 1a. We will let $T_i$ denote the temperature of Chiller $i$ and let $W_i$ denote the wattage of Heater $i$. The connection connection between wattage and heat flux is given by

$$\kappa \frac{\partial u}{\partial n} = \frac{W}{A},$$

where: $\kappa$ is a constant depending on the material of the plate; $\partial u/\partial n$ denotes the derivative of $u$ in the direction normal to (perpendicularly away from) the boundary; $W$ is the wattage of the heater; and $A$ is the surface area of the contact between the slab and the heater.

## Exercises

**Exercise 1.** Formulate a boundary value problem that models the eventual temperature in the plate at a depth of $d/2$ units. Find four "subproblems," each with only a single inhomogeneous boundary condition along one edge, whose solutions sum to give the solution of the first problem.

**Exercise 2.** Solve the four "subproblems" of Exercise 1. Your answers will include the variables $a$, $b$, $d$, $T_i$, $W_i$ and $\kappa$. The sum of these four solutions gives the analytic expression for the temperature that we sought.

**Exercise 3.** Given $M \in \mathbb{N}$, let $\Delta x = a/M$ and $\Delta y = b/M$. Create a Maple worksheet which approximates the complete solution to the boundary value problem of Exercise 1 at the center of the plate, as well as at the sample points shown in Figure 1b, and stores these values in an Excel file. It is required that:

- The values for $a$, $b$, $d$, $T_i$, $W_i$, $\kappa$ and $M$ are stored in variables.

- The number of terms $N$ in the partial sums approximating the series solution is stored in a variable.

- Your code is carefully documented and written so that the variables above are easily identified and modified.

**Exercise 4.** Figure 2 shows temperature data (in °C) obtained from an experimental set-up using a $12'' \times 12'' \times 2''$ aluminum slab. Use your Maple worksheet (with $M = 4$) and the fact that $\kappa \approx 167\,\mathrm{W/m}\,°\mathrm{C}$ for aluminum to compare the analytic solution with the actual laboratory measurements in each case. Discuss your findings.

## Instructions

- You may work in a group of up to 4 students. Each student in a group is expected to contribute equally to the project. If you feel that one of your group members is not doing his share, please let me know.

- Your written solutions to the exercises must be typed (this may be negotiable) and submitted as a PDF file. Diagrams produced using Maple must be included in the body of this document. The file you submit should be named using the convention `Project2_`*lastname(s)*`.pdf`.

**Test 1: Watt 1 = 50.4 [W]; Watt 2 = 50.3 [W]**

Experimental Data (C°)

|  | TC0 | Tavg | | TC1 |
|---|---|---|---|---|
|  | 19.5 | 19.5 | | 19.4 |
| | TC4 | TC5 | TC6 | TC7 |
| TC2 | 18.7 | 19.6 | 20.1 | 20.4 |
| 18.6 | TC8 | TC9 | TC10 | TC11 |
| Tavg | 19.4 | 20.4 (TC12 21.9) | 21.6 | 22.9 |
| 18.8 | TC13 | TC14 | TC15 | TC16 |
| TC3 | 20.5 | 21.0 | 23.1 | 24.7 |
| 19.0 | TC17 | TC18 | TC19 | TC20 |
| | 21.3 | 23.2 | 24.6 | 26.2 |

Watt 1 / Watt 2

**Test 2: Watt 1 = 101.7 [W]; Watt 2 = 102.1 [W]**

Experimental Data (C°)

|  | TC0 | Tavg | | |
|---|---|---|---|---|
|  | 22.3 | 22.5 | | 22.6 |
| | TC4 | TC5 | TC6 | TC7 |
| TC2 | 20.6 | 22.4 | 23.6 | 24.5 |
| 20.0 | TC8 | TC9 | TC10 | TC11 |
| Tavg | 21.9 | 24.0 (TC12 27.0) | 26.6 | 29.3 |
| 20.3 | TC13 | TC14 | TC15 | TC16 |
| TC3 | 23.7 | 25.1 | 29.5 | 32.5 |
| 20.6 | TC17 | TC18 | TC19 | TC20 |
| | 25.3 | 29.3 | 32.3 | 35.2 |

Watt 1 / Watt 2

**Test 3: Watt 1 = 100.3 [W]; Watt 2 = 49.8 [W]**

Experimental Data (C°)

|  | TC0 | Tavg | | TC1 |
|---|---|---|---|---|
|  | 21.3 | 21.5 | | 21.6 |
| | TC4 | TC5 | TC6 | TC7 |
| TC2 | 20.0 | 21.5 | 22.4 | 23.2 |
| 19.5 | TC8 | TC9 | TC10 | TC11 |
| Tavg | 20.8 | 22.6 (TC12 24.8) | 24.7 | 27.2 |
| 19.7 | TC13 | TC14 | TC15 | TC16 |
| TC3 | 22.1 | 23.3 | 26.5 | 29.3 |
| 19.8 | TC17 | TC18 | TC19 | TC20 |
| | 22.7 | 25.7 | 28.2 | 30.9 |

Watt 1 / Watt 2

**Test 4: Watt 1 = 50.9 [W]; Watt 2 = 102.1 [W]**

Experimental Data (C°)

|  | TC0 | Tavg | | TC1 |
|---|---|---|---|---|
|  | 21.3 | 21.4 | | 21.5 |
| | TC4 | TC5 | TC6 | TC7 |
| TC2 | 20.0 | 21.4 | 22.2 | 22.6 |
| 19.6 | TC8 | TC9 | TC10 | TC11 |
| Tavg | 21.0 | 22.8 (TC12 24.8) | 23.4 | 25.8 |
| 19.9 | TC13 | TC14 | TC15 | TC16 |
| TC3 | 22.6 | 23.7 | 26.5 | 28.4 |
| 20.2 | TC17 | TC18 | TC19 | TC20 |
| | 24.1 | 27.2 | 29.2 | 31.0 |

Watt 1 / Watt 2

Figure 2: Experimental data

- Your Maple code must be submitted as a separate worksheet file, named using the convention Project2_*lastname(s)*.mw.

- Your PDF and Maple documents must be uploaded to http://tlearn.trinity.edu by 5pm on April 18. Please only upload one copy of each document per group. Late projects *will not be accepted.*

- Failure to adhere to these guidelines will be penalized. If you have any questions or concerns, please ask me.