Exercise 1. Complete the "inhomogeneous Neumann" example from class by solving the heat problem

$$u_t = \frac{1}{4}u_{xx}, \quad 0 < x < 1, \ 0 < t,$$
$$u_x(0, t) = u_x(1, t) = 0, \quad 0 < t,$$
$$u(x, 0) = -\frac{3}{2}x^2 + 5x, \quad 0 < x < 1.$$

Exercise 2. Textbook exercise 3.6.4 [*Suggestion:* Use the result of exercise 2.3.6.]

Exercise 3. Repeat the preceding exercise, replacing the given boundary conditions with

$$u_x(0, t) = -1 \quad \text{and} \quad u_x(L, t) = 1 \quad \text{for } t > 0.$$

Use your solution to approximate the time when the minimum temperature in the bar is 60.

Exercise 4. If $\tan(\mu L) = -\mu/\kappa$, show that

$$\int_0^L \sin^2(\mu x)\, dx = \frac{\kappa L + \cos^2(\mu L)}{2\kappa}.$$

Exercise 5. Textbook exercise 3.6.10

Exercise 6. Textbook exercise 3.6.13