Introduction to Abstract Mathematics — Spring 2017

Assignment 3.1 — Due February 3

**Remark:** Throughout these exercises the word "implies" is used in the meta-language sense, and should not be confused with the symbol $\Rightarrow$.

**Exercise 1.** Let $H$ and $C$ be (compound symbolic) statements. Show that $H$ implies $C$ if and only if $H \Rightarrow C$ is a tautology.

**Exercise 2.** [*Proof by contradiction*] Let $H$, $C$ and $X$ be statements. Suppose that $X$ is a contradiction (i.e. $X$ is always false). Show that if $H \wedge (\neg C)$ implies $X$, then $H$ implies $C$.

**Exercise 3.** Let $A$, $B$ and $C$ be symbolic statements built from statement variables and logical connectives. Show that if $A$ implies $B$, then $B \Rightarrow C$ implies $A \Rightarrow C$.

**Exercise 4.** Prove that if $n$ as an even integer, then so is $n^2$.

**Exercise 5.** Suppose that $a$ and $b$ are real numbers. Prove that if $a < b$, then $a < \frac{a+b}{2} < b$.

**Exercise 6.** Textbook exercises 3.1.12 and 3.1.15