



**Exercise 1.** Prove that if  $m$  and  $n$  are consecutive integers, then exactly one of them is even and the other is odd.

**Exercise 2.**(Arithmetic Mean-Geometric Mean Inequality) Prove that if  $a$  and  $b$  are distinct nonnegative real numbers, then

$$\sqrt{ab} < \frac{a+b}{2},$$

i.e. that the geometric (multiplicative) mean is always less than the arithmetic (additive) mean.

**Exercise 3.** Suppose that  $x$  and  $y$  are real numbers. Prove that if  $x^2y = 2x + y$ , then  $y \neq 0$  implies  $x \neq 0$ .

**Exercise 4.** Suppose that  $x$  and  $y$  are real numbers, that  $x + y = 2y - x$ , and that  $x$  and  $y$  aren't both zero. Prove that  $y \neq 0$ .