Introduction to Abstract Mathematics — Assignment 3.3

Spring 2017 — Due February 3

Exercise 1. Find a contradiction-free proof that $x^2 = 4y + 2$ has no integer solutions.

Exercise 2. Let a, b and c be integers. Show that if $a^2 + b^2 = c^2$, then a or b is even.

Exercise 3. Show that $x^2 = 4y + 3$ has no integer solutions.

Exercise 4. Show that the polynomial $x^3 + x + 1$ does not have rational roots.

Exercise 5. Euclid's proof that there are infinitely many prime numbers (p. 4) is presented as a proof by contradiction, but it really isn't. What Euclid's technique actually establishes is that if there are at least n prime numbers, then there are at least $n + 1$ of them. Why does this show that there are infinitely many prime numbers?