Introduction to Abstract Mathematics — Assignment 4.1
Fall 2013 — Due February 10

**Exercise 1.** Conjecture and prove a formula for the sum of the first $n$ Fibonacci numbers.

**Exercise 2.** For $n \geq 1$, the $n$th *harmonic number* is defined to be

$$H_n = \sum_{k=1}^{n} \frac{1}{k}.$$

Prove that for all $n \geq 0$, $H_{2^n} \geq 1 + \frac{n}{2}$.

**Exercise 3.** Show that for any integer $n \geq 1$,

$$2\cos\frac{\pi}{2^{n+1}} = \underbrace{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\cdots+\sqrt{2}}}}}}_{n \text{ twos}}.$$

[*Suggestion:* Use the half angle formula $2\cos^2\theta = 1 + \cos 2\theta$.]

**Exercise 4.** What is wrong with the following "proof" that all M&Ms have the same color?

> We will prove that all the M&Ms in any (finite) group of M&Ms have a single color, which implies the result. To do so we induct on $n$, the number of M&Ms in a group. If $n = 1$, then since every M&M has the same color as itself, the result is true. Now suppose that, for some $n$, we have proven that every M&M in a group of $n$ M&Ms has the same color. Consider a group of $n+1$ M&Ms. By the inductive hypothesis, the first $n$ M&Ms in this group all have the same color, and the last $n$ also all have the same color. Since the $n$th M&M is in both groups, all $n+1$ M&Ms must have its color, and hence the whole group has a single color. By induction, this completes the proof.