Exercise 1. What amounts of money can be formed using $2 and $5 bills? Be sure to prove your answer is correct!

Exercise 2. Let $x \neq 0$ be a real number and suppose that $x + 1/x$ is an integer. Prove that $x^n + 1/x^n$ is an integer for all $n \geq 1$.

Exercise 3. Prove that for any sequence $a_1, a_2, \ldots, a_{2n+1}$ of real numbers satisfying

$$a_1 \geq a_2 \geq \cdots \geq a_{2n+1},$$

one has

$$a_1^2 - a_2^2 + a_3^2 - \cdots + a_{2n+1}^2 \geq (a_1 - a_2 + a_3 - \cdots + a_{2n+1})^2.$$

[*Suggestions:* Induct on n and prove both the $n = 0$ and $n = 1$ cases directly. Then (cleverly) use the $n = 1$ case during the inductive step.]