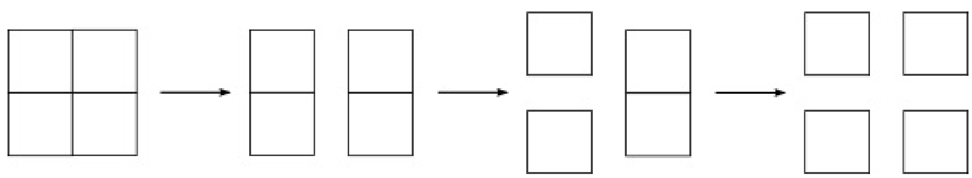
Introduction to Abstract Mathematics — Assignment 5.1
Spring 2017 — Due February 17

Exercise 1. Suppose you are given a rectangular chocolate bar made up of $m \times n$ squares of chocolate ($m, n \geq 1$). Your task is to divide it into mn individual squares by breaking it (or any of the intermediate pieces) along any of its horizontal or vertical perforations. An example in the 2×2 case is shown below.



Prove that no matter how you choose to break the chocolate bar, it will always require exactly $mn - 1$ breaks.

Exercise 2. Prove that for any $m, n \in \mathbb{N}_0$ one has

$$F_{m+n} = F_m F_{n+1} + F_{m-1} F_n,$$

where F_k denotes the kth Fibonacci number (recall that we defined $F_{-1} = 1$).[1] [*Suggestion:* As we discussed in class, begin by choosing an arbitrary $m \geq 0$, and then induct on n.]

Exercise 3. Let F_n denote the nth Fibonacci number. Prove that for all $m, n \geq 1$, if m divides n, then F_m divides F_n. [*Suggestion:* Instead prove that F_m divides F_{km} for all $k, m \geq 1$ (why is this equivalent?). Take advantage of Exercise 2.]

[1] More generally, one can show that $F_n = F_a F_b + F_c F_d$ whenever $n = a + b - 1 = c + d + 1$.