Introduction to Abstract Mathematics — Assignment 6.1
Spring 2017 — Due February 27

Exercise 1. Suppose that T is a nonempty subset of $\mathbb{Z}$ that is *bounded below*, i.e. there is an integer m so that $m \leq n$ for all $n \in T$. Show that T has a least element. [*Suggestion:* Add $1-m$ to every element of T to obtain a nonempty subset of $\mathbb{N}$, and apply the Well-Ordering Principle.]

Exercise 2. Prove that for all $n \geq 1$,

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots + \frac{1}{2n-1} - \frac{1}{2n} = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \cdots + \frac{1}{2n}.$$

Exercise 3. For $n \geq 1$ let H_n denote the nth harmonic number, as defined in exercise 4.1.2. Prove that for all $n \geq 2$,

$$\sum_{k=1}^{n-1} H_k = nH_n - n.$$