Introduction to Abstract Mathematics — Assignment 7.1
Spring 2017 — Due March 3

Exercise 1. Let $a, b, c \in \mathbb{Z}$. Verify the following properties of divisibility.

a. Every integer $n \neq 0, \pm 1$ is divisible by a prime.

b. If $a|b$ and $a|c$, then $a|(xb + yc)$ for all $x, y \in \mathbb{Z}$.

c. If $a|b$ and $a|b+1$, then $a = \pm 1$.

d. If $a|b$ and $b|c$, then $a|c$.

Exercise 2. Let S be a subset of $\mathbb{Z}$ and suppose S is *bounded above*, i.e. there exists an $M \in Z$ so that $n \leq M$ for all $n \in S$. Prove that if S is nonempty, then S has a greatest element. [*Suggestion:* Consider the set $-S = \{-n \,|\, n \in S\}$ and apply exercise 6.1.1.]

Exercise 3. Show that there are infinitely many integers that are divisible by 5 but leave a remainder of 2 when divided by 3.