PARTIAL DIFFERENTIAL EQUATIONS
SPRING 2015

ASSIGNMENT 3.3
DUE FEBRUARY 2

Exercise 1. Textbook exercise 2.1.15 [*Suggestion:* Consider $F(x + 2\pi) - F(x)$.]

Exercise 2. Textbook exercises 2.2.1–2.2.4

Exercise 3. Let

$$\begin{aligned} \mathbf{b}_1 &= (1, 1, 0, 1), \\ \mathbf{b}_2 &= (-1, 3, 1, -2), \\ \mathbf{b}_3 &= (-1, 0, 1, 1), \\ \mathbf{b}_4 &= (-2, 1, -3, 1). \end{aligned}$$

a. Show that the vectors $\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$ and $\mathbf{b}_4$ form an orthogonal basis for $\mathbb{R}^4$.

b. Let $\mathbf{x} = (1, -2, 3, -4)$. Express $\mathbf{x}$ as a linear combination of $\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3, \mathbf{b}_4$.

c. Repeat part **b** for the vector $\mathbf{y} = (2, 1, 0, 3)$.

d. Repeat part **b** for the vector $\mathbf{z} = (a, b, c, d)$.

The exercises below make use of the following definition. Given (integrable) functions f and g on the interval $[a, b]$, we define their *inner product* to be

$$\langle f, g \rangle = \int_a^b f(x)g(x)\,dx.$$

We say f and g are *orthogonal (on $[a, b]$)* if $\langle f, g \rangle = 0$.

Exercise 4. Show that if $\alpha, \beta \in \mathbb{R}$ and f, g, h are functions on $[a, b]$, then $\langle f, g \rangle = \langle g, f \rangle$ and

$$\langle \alpha f + \beta h, g \rangle = \alpha \langle f, g \rangle + \beta \langle h, g \rangle,$$

i.e. the inner product is symmetric and is linear in its first (hence second) variable.

Exercise 5. Let $p_1(x) = 3x^2 - x - 1$, $p_2(x) = 5x^2 - 5x + 1$ and $p_3(x) = 10x^2 - 16x + 3$.

a. Show that p_1, p_2 and p_3 are pairwise orthogonal on the interval $[0, 1]$.

b. Because they are orthogonal on $[0, 1]$, the polynomials p_1, p_2 and p_3 form a basis for the vector space of all polynomials of degree ≤ 2. Emulate the inner product procedure used for vectors to express $p(x) = x^2 + x + 1$ as a linear combination of p_1, p_2 and p_3.

Exercise 6. Show that any (integrable) even function is orthogonal to any (integrable) odd function on $[-a, a]$.