

$\begin{array}{c} {\rm Number\ Theory\ I} \\ {\rm Spring\ 2018} \end{array}$

Exercise 1. Find a generator $g + 37\mathbb{Z}$ of $(\mathbb{Z}/37\mathbb{Z})^{\times}$ so that $g + 37^2\mathbb{Z}$ does not generate $(\mathbb{Z}/37^2\mathbb{Z})^{\times}$. [Suggestion: Use a computer.]

Exercise 2. For each $n \geq 1$, find a generator of $(\mathbb{Z}/3^n\mathbb{Z})^{\times}$.

Exercise 3. Express each of the congruence classes $1 + 16\mathbb{Z}$, $3 + 16\mathbb{Z}$, $5 + 16\mathbb{Z}$, \ldots , $13 + 16\mathbb{Z}$, $15 + 16\mathbb{Z}$ in the form $\pm 5^k + 16\mathbb{Z}$ for some $k \in \mathbb{N}_0$.