Exercise 1. Let $G_1, G_2, \ldots, G_n$ be abelian groups and set $G = G_1 \times G_2 \times \cdots G_n$. Then it is not hard to show that G is also an abelian group under coordinate-wise operations. Prove that

$$G(2) = G_1(2) \times G_2(2) \times \cdots G_n(2).$$

Exercise 2. In the proof of Gauss' generalization of Wilson's Theorem that we gave today, we paired together elements of $(\mathbb{Z}/n\mathbb{Z})^\times$ of the form $a + n\mathbb{Z}$ and $-a + n\mathbb{Z}$. This implicitly required us to know that $a + n\mathbb{Z} \neq -a + n\mathbb{Z}$ (otherwise we wouldn't have an actual pair of elements). Prove that if $n \geq 3$, then this is always the case.

Exercise 3. The fact that $|G(2)| = 2$ for a cyclic group G of even order is, naturally, a specialization of a more general fact: if G is a cyclic group of order n, $d|n$ and $G(d) = \{g \in G \mid g^d = e\}$, then $|G(d)| = d$. Emulate the proof we gave for $d = 2$ to prove this for $d = 3$.