NUMBER THEORY I
SPRING 2018

ASSIGNMENT 11.1
DUE APRIL 11

Exercise 1. Let $B \geq 2$ and consider the following algorithm. Let $n \in \mathbb{N}_0$ and successively use the division algorithm to write

$$\begin{aligned} n &= q_0 B + a_0, \;\; 0 \leq a_0 < B, \\ q_0 &= q_1 B + a_1, \;\; 0 \leq a_1 < B, \\ q_1 &= q_2 B + a_2, \;\; 0 \leq a_2 < B, \\ &\vdots \\ q_i &= q_{i+1} B + a_{i+1}, \;\; 0 \leq a_{i+1} < B, \\ &\vdots \end{aligned}$$

a. Show that $q_0 > q_1 > q_2 > \cdots \geq 0$. Conclude that eventually $q_m = 0$ and the algorithm terminates with an equation of the form $q_{m-1} = 0 \cdot B + a_m$.

b. Show that

$$n = a_m B^m + a_{m-1} B^{m-1} + \cdots + a_1 B + a_0.$$

[*Suggestion:* Back-substitute.]

Exercise 2.

a. Express the message unit ZETA FUNCTION as an integer by writing it as a base 27 expansion. [*Remark:* The result has 19 (decimal) digits.]

b. Express the integer $M = 4048645098839$ as a nine letter message unit by writing it in base 27.