Number Theory I
Spring 2018

Assignment 13.1
Due April 25

Exercise 1. Consider the quadratic congruence

$$x^2 - 7x + 2 \equiv 0 \pmod{n}. \tag{1}$$

a. If $n = 4102925927536873$, how many solutions $\pmod{n}$ does (1) have?

b. If $n = 5211824826871163$, how many solutions $\pmod{n}$ does (1) have?

Exercise 2. Find every solution (mod 15015) to the quadratic congruence $x^2 + 111x + 5 \equiv 0 \pmod{15015}$. [*Suggestion:* For each prime power p^m dividing 15015, solve $x^2 + 111x + 5 \equiv 0 \pmod{p^m}$ by hand. Then use a computer to combine these solutions in every possible way using the CRT.]

Exercise 3. Prove the following generalization of Euler's Criterion. If G is a finite cyclic group whose order is divisible by k, then $a \in G$ has a kth root in G if and only if $a^{|G|/k} = e$.