NUMBER THEORY I
SPRING 2018

ASSIGNMENT 3.1
DUE JANUARY 31

Exercise 1. Prove that every perfect cube is of the form $7k$ or $7k \pm 1$ for some $k \in \mathbb{Z}$.

Exercise 2. Prove that $4^n \equiv 3n + 1 \pmod 9$ for any $n \in \mathbb{N}$. [*Suggestion:* Write $4 = 3 + 1$ and use the Binomial Theorem.]

Exercise 3. Prove that $53^{103} + 103^{53}$ is divisible by 39 and that $111^{333} + 333^{111}$ is divisible by 7.

Exercise 4. Show that if $p > 3$ is prime, then 13 divides $10^{2p} - 10^p + 1$. [*Suggestion:* Show first that $p \equiv \pm 1 \pmod 6$, then compute $10^6 \pmod{13}$.]