Number Theory I
Spring 2018

Assignment 4.1
Due February 7

Exercise 1. Let R be a ring with additive identity element 0. Prove that for all $a \in R$, $a0 = 0a = 0$. [*Suggestion:* Compute $0 + 0$ and multiply both sides of your result by a.]

Exercise 2. Compute the following inverses.

a. $(19 + 23\mathbb{Z})^{-1}$ in $\mathbb{Z}/23\mathbb{Z}$

b. $(43 + 103\mathbb{Z})^{-1}$ in $\mathbb{Z}/103\mathbb{Z}$

c. $(17 + 299\mathbb{Z})^{-1}$ in $\mathbb{Z}/299\mathbb{Z}$

Exercise 3.

a. Let R be a ring and $a \in R^\times$. Show that the function $L_a : R \to R$ given by $x \mapsto ax$ is a bijection.

b. Let G be a group and $g \in G$. Show that the function $L_g : G \to G$ given by $x \mapsto gx$ is a bijection.

Exercise 4. Let $n \in \mathbb{N}$ and $a \in \mathbb{Z}$ with $(a, n) = 1$. Suppose that $S \subseteq \mathbb{Z}$ contains exactly one element from each congruence class in $\mathbb{Z}/n\mathbb{Z}$. Prove that the same is true of $aS = \{as \mid s \in S\}$. [*Suggestion:* Apply exercise **3a** to the ring $\mathbb{Z}/n\mathbb{Z}$.]