



NUMBER THEORY I
SPRING 2018

ASSIGNMENT 4.2
DUE FEBRUARY 7

Exercise 1. Prove that for any $n \in \mathbb{N}$, the fraction $(12n + 1)/(30n + 2)$ is reduced.

Exercise 2. Let $a, b \in \mathbb{N}$. Prove that $(2^a - 1, 2^b - 1) = 2^{(a,b)} - 1$.

Exercise 3. Show that the solution set of the *quadratic congruence*

$$x^2 \equiv 1 \pmod{35}$$

consists of exactly 4 congruence classes modulo 35.

Exercise 4. Let $a, b \in \mathbb{Z}$ and set $d = (a, b)$. Write $a = a'd$ and $b = b'd$ for some $a', b' \in \mathbb{Z}$. Prove that if $d \neq 0$, then $(a', b') = 1$.