NUMBER THEORY I
SPRING 2018

ASSIGNMENT 5.1
DUE FEBRUARY 14

Exercise 1. Solve the linear congruence, if possible. Feel free to use a computer to find GCDs and modular inverses.

a. $377x \equiv 300 \pmod{479}$

b. $212x \equiv 136 \pmod{872}$

c. $885x \equiv 217 \pmod{903}$

d. $57x \equiv 66 \pmod{753}$

Exercise 2. Let $a, b \in \mathbb{Z}$, $n \in \mathbb{N}$. Suppose that $d \in \mathbb{N}$ is a common divisor of a, b and n. Prove that

$$a \equiv b \pmod{n} \quad \Leftrightarrow \quad \frac{a}{d} \equiv \frac{b}{d} \left(\operatorname{mod} \frac{n}{d}\right).$$

Exercise 3. Let

$$\pi_1(x) = \#\{p \leq x \,|\, p \equiv 1 \pmod 4\} \quad \text{and} \quad \pi_3(x) = \#\{p \leq x \,|\, p \equiv 3 \pmod 4\}.$$

Prove that

$$\lim_{x \to \infty} \frac{\pi_3(x)}{\pi(x)} = \frac{1}{2} \iff \lim_{x \to \infty} \frac{\pi_1(x)}{\pi(x)} = \frac{1}{2}.$$

Exercise 4. Let $a \in \mathbb{Z}$, $b \in \mathbb{N}$. Prove that if $(a, b) > 1$, then $\{p \equiv a \pmod b \,|\, p \text{ is prime}\}$ is finite.