



NUMBER THEORY I
SPRING 2018

ASSIGNMENT 5.3
DUE FEBRUARY 14

Exercise 1. Let $n \in \mathbb{N}$ and $a \in \mathbb{Z}$ with $(a, n) = 1$. Show that $a + n\mathbb{Z}$ has (additive) order n in $\mathbb{Z}/n\mathbb{Z}$. [*Suggestion:* Show that if $k(a + n\mathbb{Z}) = 0 + n\mathbb{Z}$, then $n|k$.]

Exercise 2. Find the order of every element of $(\mathbb{Z}/n\mathbb{Z})^\times$ for $n = 7, 8, 9, 10$.

Exercise 3. If G is a group and $g \in G$ has order n , show that $e, g, g^2, \dots, g^{n-1}$ are distinct elements of G . Also show that *any* power of g is equal to one of these. [*Suggestions:* For the first part argue by contradiction. For the second apply the division algorithm to the exponent.]