NUMBER THEORY I
SPRING 2018

ASSIGNMENT 6.1
DUE FEBRUARY 21

Exercise 1.

a. Compute the last two digits of 3^{45}. [*Hint:* $\varphi(100) = 40$.]

b. Find the remainder when 2^{100000} is divided by 77. [*Hint:* $\varphi(77) = 60$.]

Exercise 2. If m, $n \in \mathbb{N}$ are relatively prime, prove that

$$m^{\varphi(n)} + n^{\varphi(m)} \equiv 1 \pmod{mn}.$$

[*Suggestion:* Argue that it suffices to show the stated congruence holds modulo m and modulo n separately.]

Exercise 3.

a. Verify that $4(29!) + 5!$ is divisible by 31.

b. Show that $18! \equiv -1 \pmod{437}$.

Exercise 4. Prove that if $n > 4$ is composite, then $(n-1)! \equiv 0 \pmod{n}$. [*Suggestion:* Use the fact that $n = ab$ with $1 < a, b < n$. The case $a = b$ needs to be treated separately.]