Number Theory I
Spring 2018

Assignment 7.1
Due February 28

Exercise 1. Solve the simultaneous congruences

$$x \equiv 5 \pmod{11},$$
$$x \equiv 14 \pmod{29},$$
$$x \equiv 15 \pmod{31}.$$

Exercise 2. Solve the simultaneous congruences

$$2x \equiv 1 \pmod{5},$$
$$3x \equiv 9 \pmod{6},$$
$$4x \equiv 1 \pmod{7},$$
$$5x \equiv 9 \pmod{11.)}$$

[*Suggestion:* Solve each congruence for x first by multiplying by the appropriate inverse, then use the CRT.]

Exercise 3. Regiomontanus (1436–1476) asked for the smallest positive integer leaving remainders of 3, 11 and 15 when divided by 10, 13 and 17, respectively. Find this integer.

Exercise 4. Find the inverse of $17+210\mathbb{Z}$ by finding the inverses of $17+2\mathbb{Z}$, $17+3\mathbb{Z}$, $17+5\mathbb{Z}$ and $17+7\mathbb{Z}$, then using the Chinese remainder theorem to "glue" these results together.