Number Theory I
Spring 2018

Assignment 8.2
Due March 7

Exercise 1. Prove that

$$n = 185327073637076855291755518012905992650368428443467865444249246547055030415\\ 530205721466814637573922492124921069090395720566635195412142549271807948869$$

is composite.

Exercise 2. Prove that 561 is a Carmichael number. [*Suggestion:* Show that if $(a, 561) = 1$, then $a^{560} \equiv 1 \pmod{p}$ for each prime $p | 561$. Why is this sufficient?]

Exercise 3.

a. Let $a, b \in \mathbb{R}$ and set $t = a + b$, $n = ab$. Show that if you are given t and n, then you can determine a and b. [*Suggestion:* Consider the roots of the polynomial $x^2 - tx + n$.]

b. Let p, q be distinct primes and set $n = pq$. Show that if you are given n and $\varphi(n)$, then you can determine p and q. [*Suggestion:* Express $p + q$ in terms of n and $\varphi(n)$ and use part **a**.]

c. Given that $n = 3452366617984213531848875693960104104481$ is the product of two distinct primes and $\varphi(n) = 3452366617984213531729007293807679518912$, factor n.

Exercise 4. Let $B \geq 2$ and consider the base B expansion of $n \in \mathbb{N}$:

$$n = \sum_{j=0}^{N} d_j B^j, \quad d_j \in \{0, 1, 2, \ldots, B-1\}, \quad d_N \neq 0.$$

The *place-value notation* for this expansion is $(d_N d_{N-1} \cdots d_1 d_0)_B$.[1]

a. Express 83177 in base 7 place-value notation.

b. The base 16 expansion of a natural number is referred to as its *hexadecimal* representation. When using hexadecimal expansions it is typical to represent the "digits" 10, 11, 12, 13, 14 and 15 by the symbols A, B, C, D, E and F, respectively.

i. Express $(9EF6)_{16}$ in decimal notation.

ii. Express 27999 in hexadecimal place-value notation.

[1] When $B = 10$, we omit the subscript and parentheses, obtaining the familiar digital expansion of n.